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STRESSES IN A PERFORATED CYLINDRICAL SHELL*

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Abstract—It is assumed that the deformations of a thin cylindrical elastic shell, subjected to a uniform lateral pressure, are adequately described by the Donnell shell equations. The surface of the shell is perforated by a rectangular cutout. The tangential membrane displacements and the normal membrane stresses vanish on the simply supported edges of the hole and the ends of the cylinder. Numerical solutions are obtained by replacing the boundary value problem by a finite difference approximation and solving the resulting system of linear algebraic equations by an iteration procedure.

1. INTRODUCTION

THE presence of a hole in the wall of a cylindrical elastic shell that is subjected to surface and edge forces, significantly alters the distribution and magnitudes of the stresses. It also complicates the mathematical analysis of the shell problem. Approximate methods of solution, which are restricted in their applicability, have been previously proposed† [1–6].

In this paper, a numerical method is presented to determine the stresses and displacements in the shell for a specific cutout problem. However, with suitable modifications, it can be extended to include a variety of cutout shapes and other boundary and loading conditions.

The procedure we employ is the well-known finite difference method. In this way, the boundary value problem of shell theory is approximated by a system of linear algebraic equations. In principle, there is usually no difficulty in the application of the method. However, for cutout problems, in which the solutions vary rapidly, many mesh points may be required. Roundoff errors and excessive computing time make it impractical to use the "standard" methods (e.g. Gauss elimination) to solve the resulting large system of algebraic equations. We present and successfully apply an iterative method involving two "acceleration" parameters to solve the algebraic equations. The procedure is described in Section 3 and the computational methods required to apply it are outlined in Section 4.

For the specific problem investigated the perforation is rectangular, i.e. the edges of the cutout coincide with generators and parallel circles of the cylinder's midsurface. The shell is loaded by a uniform pressure applied normal to the surface. On the simply supported boundaries the tangential membrane displacements and the normal membrane stresses vanish. It is assumed that the deformations are adequately described by the Donnell thin shell theory [7, 8].

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† Reference [4] contains a general discussion of shell cutout problems and references to previous work.

In Section 5 the results of the computation are discussed and graphs of stresses and displacements are shown. They indicate that the stresses are large near the corner of the rectangular cutout. These high stresses may have slowed the convergence of the iterative procedure. We therefore conjecture that for problems with smoother cutouts the method may be more efficient.

2. FORMULATION

The cylindrical shell is of mean radius R, thickness t and length $2L$. A cylindrical coordinate system r, θ, z is employed where r and θ are plane polar coordinates and z is the axial distance measured from one end of the cylinder. Thus the coordinates are in the ranges $R-t/2 \leq r \leq R+t/2$, $-\pi \leq \theta \leq \pi$, $-L \leq z \leq L$. The dimensionless coordinates x and y are defined by the relations

$$\begin{aligned} x &\equiv z/L, && |x| \leq 1, \\ y &= (R/L)\theta, && |y| \leq (\pi R/L) \equiv l. \end{aligned} \tag{1}$$

The wall of the shell is perforated by a rectangular hole whose edges are on the lines $x = \pm\alpha$ and $y = \pm\beta l$ where $0 \leq \alpha, \beta < 1$. The shell is deformed by a uniform pressure p applied normal to the surface. The ends of the shell and the edges of the hole are simply supported and the normal membrane stresses and the tangential membrane displacements vanish there. The solution is therefore assumed to be symmetric with respect to the planes $x = 0$ and $y = 0$. Thus we consider the region D, see Fig. 1, which is one quarter of the cylindrical surface. Symmetry conditions must be specified along the following three boundary lines of D: $\alpha \leq x \leq 1$, $y = 0$; $0 \leq x \leq 1$, $y = l$; $x = 0$, $\beta l \leq y \leq l$.

The Donnell theory for cylindrical shells [7, 8] can be formulated as a boundary value problem for two coupled fourth order partial differential equations in the two dependent variables $W(z, \theta)$ and $F(z, \theta)$. Here, W is the normal displacement of the midsurface $r = R$ of the cylinder and EF is the membrane stress function where E is Young's modulus. The positive directions of p and W are outward from the center line of the cylinder. In dimensionless variables the differential equations of the Donnell theory are

$$\Delta^2 w + \tau \frac{\partial^2 f}{\partial x^2} = 1, \qquad \Delta^2 f = \tau \frac{\partial^2 w}{\partial x^2} \tag{2a}$$

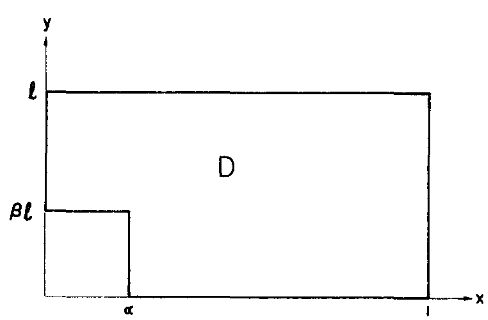

FIG. 1. The domain of integration, D, which is one quarter of the cylindrical surface.

where Δ^2 is the biharmonic operator,

$$\Delta^2 \equiv \frac{\partial^4}{\partial x^4}+2\frac{\partial^4}{\partial x^2\,\partial y^2}+\frac{\partial^4}{\partial y^4}. \tag{2b}$$

On the boundary of D we specify that

$$\begin{aligned}
&w = \frac{\partial^2 w}{\partial x^2} = f = \frac{\partial^2 f}{\partial x^2} = 0, &&\text{for } x = 1, && 0 \le y \le l,\\
&w = \frac{\partial^2 w}{\partial x^2} = f = \frac{\partial^2 f}{\partial x^2} = 0, &&\text{for } x = \alpha, && 0 \le y \le \beta l,\\
&w = \frac{\partial^2 w}{\partial y^2} = f = \frac{\partial^2 f}{\partial y^2} = 0, &&\text{for } 0 \le x \le \alpha, && y = \beta l,
\end{aligned} \tag{3a}$$

$$\begin{aligned}
&\frac{\partial w}{\partial y} = \frac{\partial^3 w}{\partial y^3} = \frac{\partial f}{\partial y} = \frac{\partial^3 f}{\partial y^3} = 0, &&\text{for } y = 0, && \alpha \le x \le 1,\\
& &&\text{and } y = l, && 0 \le x \le 1,\\
&\frac{\partial w}{\partial x} = \frac{\partial^3 w}{\partial x^3} = \frac{\partial f}{\partial x} = \frac{\partial^3 f}{\partial x^3} = 0, &&\text{for } x = 0, && \beta l \le y \le l.
\end{aligned} \tag{3b}$$

The dimensionless variables in equations (2) and (3) are defined by

$$\begin{aligned}
&w(x, y) \equiv CW(z, \theta), \qquad f(x, y) \equiv [12(1-\nu^2)]^{1/2} C\,\frac{F(z, \theta)}{t}\\
&C \equiv \frac{E}{12(1-\nu^2)pL}\left(\frac{t}{L}\right)^3, \qquad \tau \equiv [12(1-\nu^2)]^{1/2}\,\frac{L^2}{Rt},
\end{aligned} \tag{4}$$

where ν is Poisson's ratio. The membrane stresses σ_z^0, σ_θ^0 and $\sigma_{z\theta}^0$ and the corresponding dimensionless stresses Σ_x^0, Σ_y^0 and Σ_{xy}^0 are given in terms of the dimensionless stress function $f(x, y)$ by

$$\begin{aligned}
&\textstyle\sum_x^0 (x, y) \equiv A\sigma_z^0(z, \theta) = \dfrac{\partial^2 f(x, y)}{\partial y^2}, \qquad \textstyle\sum_y^0 (x, y) = A\sigma_\theta^0(z, \theta) = \dfrac{\partial^2 f(x, y)}{\partial x^2},\\
&\textstyle\sum_{xy}^0 (x, y) \equiv A\sigma_{z\theta}^0(z, \theta) = -\dfrac{\partial^2 f(x, y)}{\partial x\,\partial y}
\end{aligned} \tag{5}$$

where

$$A^{-1} \equiv [12(1-\nu^2)]^{1/2} p(L/t)^2.$$

The quantities σ_z', σ_θ' and $\sigma_{z\theta}'$, which are proportional to the bending moments, and the dimensionless bending stresses m_x, m_y and m_{xy} are given in terms of $w(x, y)$ by,

$$
\begin{aligned}
m_x(x, y) &\equiv B\sigma_z'(z, \theta) = \frac{\partial^2 w(x, y)}{\partial x^2} + \nu \frac{\partial^2 w(x, y)}{\partial y^2}, \\
m_y(x, y) &\equiv B\sigma_\theta'(z, \theta) = \frac{\partial^2 w(x, y)}{\partial y^2} + \nu \frac{\partial^2 w(x, y)}{\partial x^2}, \\
m_{xy}(x, y) &\equiv (1-\nu)B\sigma_{z\theta}'(z, \theta) = \frac{\partial^2 w(x, y)}{\partial x\, \partial y},
\end{aligned} \tag{6}
$$

where

$$
B \equiv -\frac{t}{12p}\left(\frac{t}{L}\right)^2.
$$

If the displacements of the midsurface of the shell in the z and θ directions are $U(z, \theta)$ and $V(z, \theta)$ respectively, then the dimensionless displacements $u(x, y)$ and $v(x, y)$ are defined by

$$
u(x, y) \equiv \left(\frac{R}{L}\right)\frac{U(z, \theta)}{C}, \qquad v(x, y) \equiv \left(\frac{R}{L}\right)\frac{V(z, \theta)}{C}.
$$

The membrane stress displacement relations of the Donnell theory thus reduce to [7, 8],

$$
\begin{aligned}
\Sigma_x^0 &= \frac{\tau}{1-\nu^2}\left[\frac{\partial u}{\partial x} + \nu\left(\frac{\partial v}{\partial y} + w\right)\right], \qquad \Sigma_y^0 = \frac{\tau}{1-\nu^2}\left[\frac{\partial v}{\partial y} + w + \nu\frac{\partial u}{\partial x}\right], \\
\Sigma_{xy}^0 &= \frac{\tau}{2(1+\nu)}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right).
\end{aligned} \tag{7}
$$

The relations (5), (6) and (7) and the conditions (3) imply that the required boundary and symmetry conditions* are satisfied.

The complete formulation of the boundary value problem consists of the differential equations (2) and the boundary conditions (3). The solutions depend only on the parameters α, β, l and τ. We obtain approximate solutions for fixed α, β, and l and a sequence of τ values.

3. NUMERICAL METHODS

To apply the approximate method the region D is covered with a rectilinear mesh. The mesh is of uniform spacing δ in the x and y directions. The mesh is chosen so that the boundary lines of D coincide with mesh lines. There are $M+1$ mesh lines in the x direction and $N+1$ in the y direction so that,

$$
\delta = 1/M = l/N. \tag{8}
$$

The mesh points (x_i, y_j) are defined by,

$$
x_i = i\delta, \quad y_j = j\delta, \quad i = 0, 1, \ldots, M; \qquad j = 0, 1, \ldots N, \tag{9}
$$

* The relations actually imply that the membrane displacements tangential to each boundary are constants. However, these constants are set equal to zero.

and the integers I and J are defined by,

$$x_I = \alpha, \qquad y_J = \beta l.$$

The mesh region D_δ is defined as the set of points (x_i, y_j) interior to and on the boundary of D. The mesh region D'_δ is obtained from D_δ by excluding the mesh points: (x_M, y_j), $j = 0, \ldots, N$; (x_I, y_j), $j = 0, \ldots, J$; (x_i, y_J), $i = 0, \ldots, I$.

At each point of D'_δ the solutions $w(x_i, y_j), f(x_i, y_j)$ of equations (2) and (3) are assumed to be approximated by w_{ij}, f_{ij} which satisfy the difference equations,

$$\Delta_\delta^2 w_{ij} + \tau L_x f_{ij} = 1, \qquad \Delta_\delta^2 f_{ij} - \tau L_x w_{ij} = 0, \qquad (x_i, y_j) \in D'_\delta \tag{10}$$

Here the difference operator Δ_δ^2, obtained by replacing derivatives in the differential operator (2b) by centered difference quotients, is given by

$$\Delta_\delta^2 = L_x^2 + 2L_x L_y + L_y^2. \tag{11a}$$

L_x and L_y are the "one-dimensional" centered second difference operators defined, for any mesh function* $\{g_{ij}\}$, by

$$\begin{aligned} L_x g_{ij} &= (g_{i+1,j} - 2g_{i,j} + g_{i-1,j})(\delta)^{-2}, \\ L_y g_{ij} &= (g_{i,j+1} - 2g_{i,j} + g_{i,j-1})(\delta)^{-2}. \end{aligned} \tag{11b}$$

The approximating quantities $\{w_{ij}\}$ and $\{f_{ij}\}$ must satisfy difference equivalents of the boundary conditions (3) where all derivatives are replaced by centered difference approximations. Fictitious mesh points exterior to D_δ are introduced to apply the difference equations.

Thus a system of $2[M(N+1)-(I+1)(J+1)]$ linear algebraic equations in the $2[M(N+1)-(I+1)(J+1)]$ unknowns $\{w_{ij}\}$ and $\{f_{ij}\}$ is obtained. We assume that as $\delta \to 0$ the solution of the algebraic system converges to the solution of (2) and (3). Hence for a sufficiently small δ the solution of the algebraic problem should yield a "close" approximation to the solution of the boundary value problem.

For large M and N, it is impractical to solve the algebraic system by a direct method, e.g. Gaussian elimination. Therefore, an iteration procedure is used to obtain approximate solutions of this sytem. With $w_{ij}^{(0)}$ as an initial guess, for fixed values of α, β, l and τ, a sequence of iterates $[w_{ij}^{(n)}, f_{ij}^{(n)}]$ are defined by the recursions

$$\left.\begin{aligned} \Delta_\delta^2 f_{ij}^{(n)} = \tau L_x w_{ij}^{(n)}, \qquad \Delta_\delta^2 \bar{w}_{ij}^{(n+1)} &= -\tau L_x f_{ij}^{(n)} + 1, \\ w_{ij}^{(n+1)} = \lambda \bar{w}_{ij}^{(n+1)} + (1-\lambda) w_{ij}^{(n)} & \end{aligned}\right\} \quad \begin{aligned} &n = 0, 1, \ldots, \\ &(x_i, y_j) \in D'_\delta. \end{aligned} \tag{12}$$

Each iterate must also satisfy the difference equivalents of equation (3). The acceleration parameter λ should be determined so that the iterations (12) converge as rapidly as possible.

Therefore, for each n the solutions of two algebraic systems of the form

$$\Delta_\delta^2 g_{ij}^{(n+1)} = \phi_{ij}^{(n)} \qquad (x_i, y_j) \in D'_\delta \tag{13}$$

are required where the mesh function $\{\phi_{ij}^{(n)}\}$ is either $\{\tau L_x w_{ij}^{(n)}\}$ or $\{-\tau L_x f_{ij}^{(n)} + 1\}$ depending upon whether $\{g_{ij}^{(n)}\}$ is $\{f_{ij}^{(n)}\}$ or $\{w_{ij}^{(n+1)}\}$. A line relaxation procedure, which is a simplification of the alternating direction method [9, 10] is used to obtain a crude approximation to the solution of (13). A single horizontal and a single vertical sweep of the mesh is

* A function $\{g_{ij}\}$ defined only at points of the mesh is called a mesh function.

performed instead of alternating back and forth between horizontal and vertical sweeps.* Thus the approximate solution $\{g_{ij}^{(n+1)}\}$ of equation (13) is defined as the solution of the one dimensional difference equations [10]:

$$\left.\begin{aligned}(1+\rho L_y^2)g_{ij}^{(n+1)} &= g_{ij}^{(n+\frac{1}{2})}+\rho L_y^2 g_{ij}^{(n)},\\ (1+\rho L_x^2)g_{ij}^{(n+\frac{1}{2})} &= g_{ij}^{(n)}-2\rho L_y L_x g_{ij}^{(n)}-\rho L_y^2 g_{ij}^{(n)}+\rho\phi_{ij}^{(n)}\end{aligned}\right\}\quad (x_i, y_j)\in D'_\delta, \tag{14}$$

where $\{g_{ij}^{(n+\frac{1}{2})}\}$ and $\{g_{ij}^{(n+1)}\}$ must also satisfy the appropriate difference boundary conditions. The matrices corresponding to the one dimensional difference operators in equation (14) are easily inverted by factoring. The number ρ is a second acceleration parameter and should be determined so that the iterations (12) converge as rapidly as possible.

4. COMPUTATIONAL PROCEDURES

Application of the numerical procedure requires that the mesh spacing be "small". An acceptable value of δ is determined by comparing the solutions of a sequence of test calculations with successively finer meshes. From these tests we conclude that the $\delta = \frac{1}{24}$ mesh yields sufficiently accurate solutions for the range of parameters considered. The numerical criterion for convergence is that the iterations satisfy the condition,

$$z^{(n)} \equiv \max_{(x_i,y_j)\in D'_\delta}\left\{\left|w_{ij}^{(n)}-w_{ij}^{(n-1)}\right|, \left|f_{ij}^{(n)}-f_{ij}^{(n-1)}\right|\right\} < 10^{-k} \tag{15}$$

The value of k that is employed depends on the magnitudes of the solutions. For all our calculations we used $k = 8$ or 9. The condition, $\lim_{n\to\infty} z^{(n)} = 0$ is only necessary for convergence.

It is found, starting from a given initial iterate, that the number of iterations required to satisfy equation (15) depends on the values of ρ and λ. For fixed α, β, l and τ, the optimal values λ_0 and ρ_0 are defined as those values which minimize the number of iterations necessary to satisfy equation (15). To obtain estimates for them a series of test calculations are employed. Thus for a fixed configuration, i.e. fixed α, β and l, and usually employing a coarse mesh estimates of λ_0 and ρ_0 are determined for an increasing sequence of τ values. These values are used for the finer meshes and λ_0 and ρ_0 for other values of τ are estimated by interpolating or extrapolating. In Table 1 representative values are given of the acceleration parameters that were employed.

TABLE 1. ACCELERATION PARAMETERS EMPLOYED IN THE COMPUTATION

	Shell I		Shell II	
τ	λ	ρ	λ	ρ
340	1·0	24·6	1·0	10·0
680	0·675	9·7	0·55	10·0
1020	0·30	9·7	0·25	9·6
1360	0·20	9·7	—	—
1700	0·12	9·7	0·10	9·6

* A limited number of numerical experiments indicate that the convergence rate of equation (12) essentially does not change if several sweeps are made through the mesh.

Numerical solutions are obtained for two configurations, which we call Shells I and II, and a sequence of values of $\tau = 340, 680, 1020, 1360, 1700$. Shells I and II are defined respectively by, $\alpha = 10/24$, $\beta = 10/22$, $l = 22/24$ and $\alpha = 7/24$, $\beta = 7/22$, $l = 22/24$. The initial iterate for a given τ is taken as the "converged" solution for the nearest τ for the same shell. When the iterations converge the dimensionless stresses are computed employing obvious finite difference equivalents of equations (5) and (6). It is observed that the number of iterations necessary for convergence depends on the values of the parameters α, β, l and τ. For example, with a fixed configuration the number increases rapidly as τ increases, and with fixed l and τ the number increases as α and β decrease. In all cases a relatively large number of iterations is needed to satisfy equation (15). For example using Shell I and $\tau = 340$, 492 iterations are needed, and for $\tau = 1020$, 2481 iterations are required. However, the computing time per iteration is relatively small, e.g. the above computations required approximately 13 and 65 minutes* respectively.

5. PRESENTATION OF RESULTS

Representative graphs of the dimensionless stresses and normal displacement obtained from the numerical solution for Shell I with $\tau = 1700$ are given in Figs. 2–8. In these

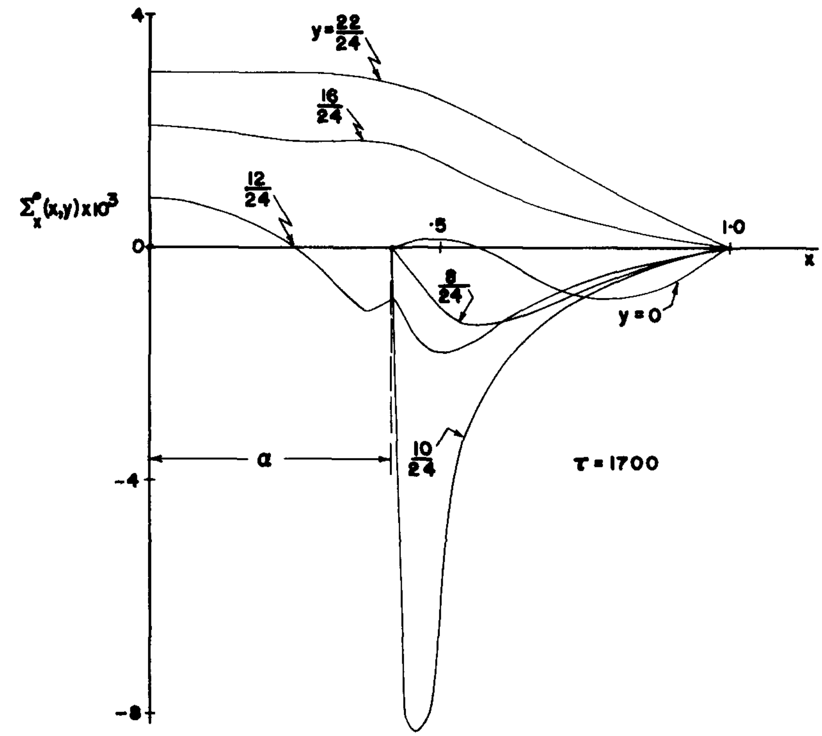

FIG. 2. The variation with the axial distance x of the dimensionless axial membrane stress Σ_x^0 for Shell I for a sequence of y values. The hole occurs at $y = \frac{10}{24}$.

* All computations employed $\nu = 0{\cdot}3$ and were performed on the IBM-7090 computer at the A.E.C. Computing and Applied Mathematics Center of the Courant Institute of Mathematical Sciences, New York University.

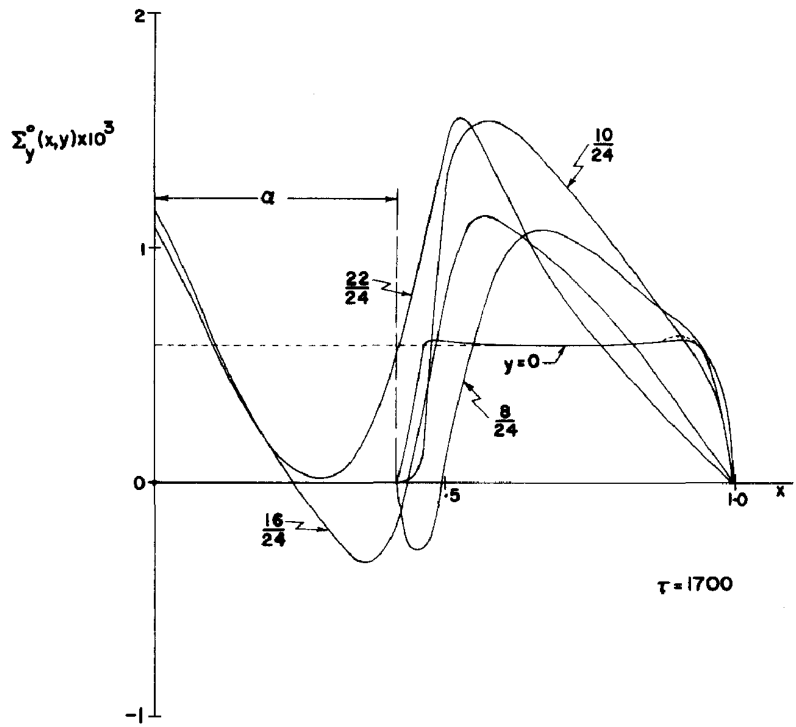

FIG. 3. The variation with x of the dimensionless hoop membrane stress Σ_y^0 for Shell I for a sequence of y values. The dotted curve is the stress if no hole is present.

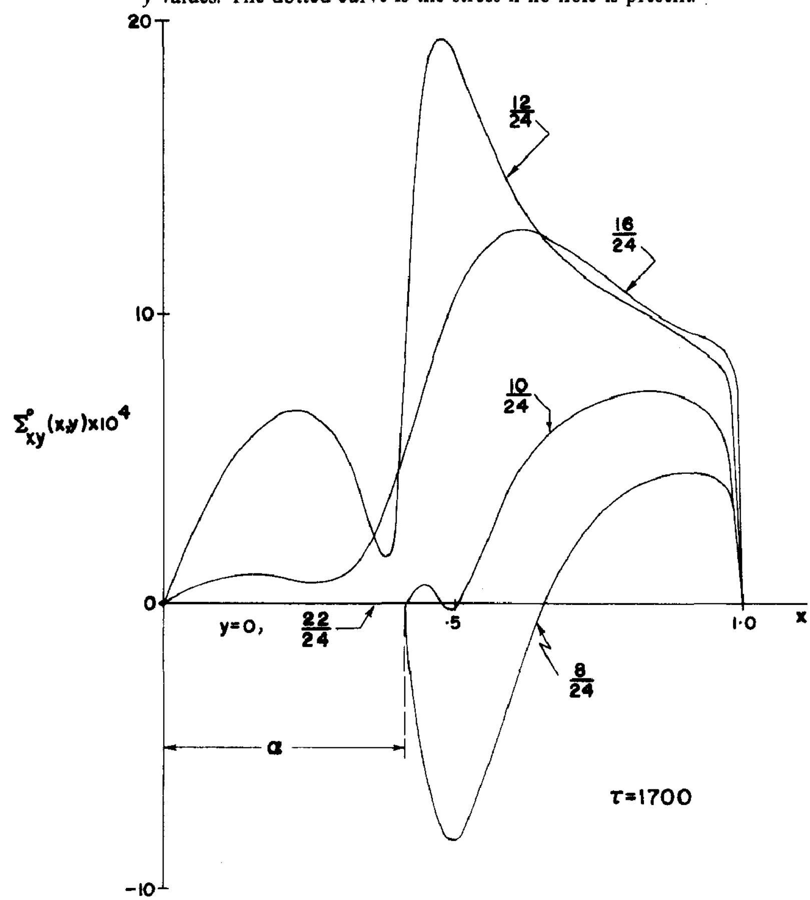

FIG. 4. The variation with x of Σ_{xy}^0 for Shell I for a sequence of y values.

figures the variations with the axial distance are shown for a sequence of y values. These results indicate that near the corner of the cutout, $x = \alpha$, $y = \beta l$, the numerical stresses are quite large. The hole apparently affects the stress distribution everywhere in the shell. The hoop membrane stress Σ_y^0 and the axial bending stress Σ_x' are influenced more by the hole than the other stresses. If $\alpha = \beta = 0$, the stresses and displacements obtained from the well-known solution of the boundary value problem are independent of y. These quantities are indicated with a super-bar and are given by,

$$\bar{w}(x) = \frac{1}{\tau^2} - \frac{\cos\beta x \cosh\beta x + \tan\beta \tanh\beta \sin\beta x \sinh\beta x}{\tau^2 \cos\beta \cosh\beta(1+\tan^2\beta \tanh^2\beta)},$$

$$\overline{\sum}_x^0(x) = \overline{\sum}_{xy}^0(x) = \overline{\sum}_{xy}'(x) \equiv 0, \qquad \overline{\sum}_y^0(x) = \tau\bar{w}(x), \tag{16}$$

$$\overline{\sum}_x'(x) = \frac{1}{\nu}\overline{\sum}_y'(x) = \frac{d^2\bar{w}}{dx^2}$$

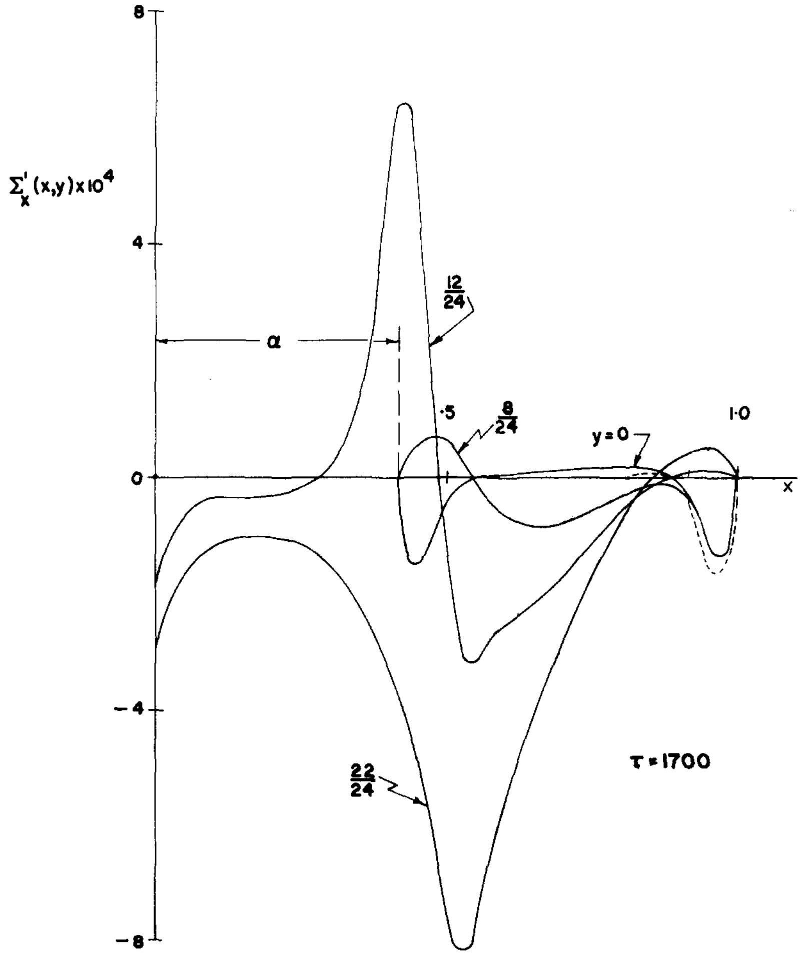

FIG. 5. The variation with x of the dimensionless axial bending stress Σ_x' ~~for Shell I for~~ a sequence of y values. For the shell with no cutout, Σ_x' coincides, within the resolution of the graph, with the x-axis for the range $0 \leq x < 0{\cdot}833$. For $0{\cdot}833 \leq x \leq 1$, Σ_x' is represented by the dotted curve.

where

$$\tau^2 = 4\beta^4.$$

These results are shown by the dotted curves in Figs. 3, 5 and 6. In Fig. 8, the lateral displacement $\bar{w}(x)$ coincides, within the resolution of the graph, with the numerical results for the line $y = 0$. These figures indicate that the cutout and its corner may increase the magnitudes of the stresses and displacements by large factors.

Although their magnitudes differ, the form of the stresses and displacements for other values of τ and for Shell II are similar to those shown in Figs. 2–8. A simple asymptotic solution is obtained for large τ by dividing the differential equations by τ and letting $\tau \to \infty$ while $w(x, y; \infty)$ and $f(x, y; \infty)$ remain finite. Integrating the resulting equations and applying the boundary conditions we obtain

$$w(x, y; \infty) = f(x, y; \infty) \equiv 0.$$

To illustrate this result, graphs are shown in Figs. 9–11 of typical stresses and displacements for Shell II on the line $y = l$ for an increasing sequence of τ values. All stresses and displacements seem to approach zero as τ increases. The decrease in the magnitudes of the solution as τ increases is another possible cause for the slow rate of convergence for large τ.

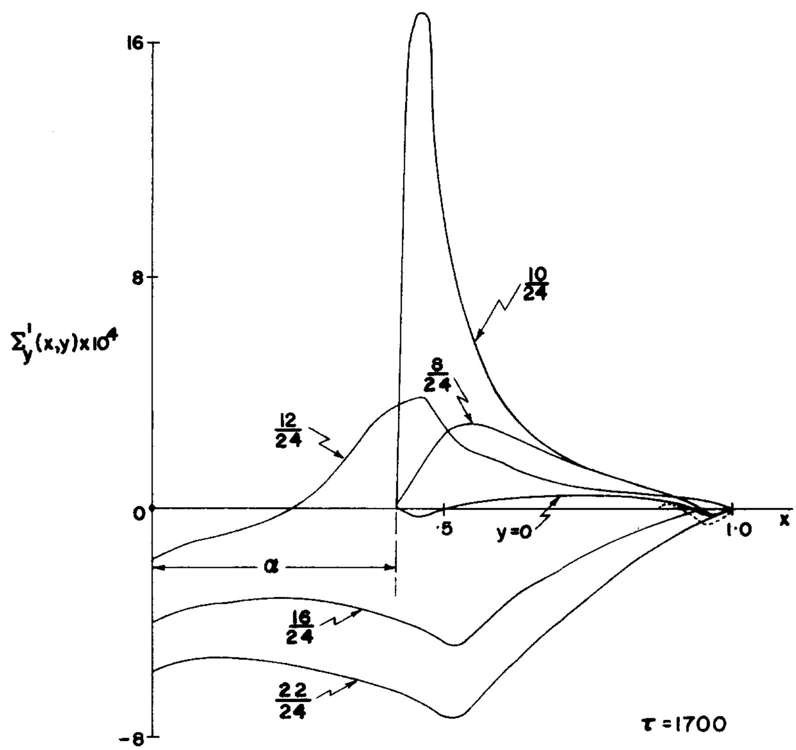

FIG. 6. The variation with x of Σ_y' for Shell I for a sequence of y values. For the shell with no cutout, $\bar{\Sigma}_y'$ coincides, within the resolution of the graph, with the x-axis for the range $0 \leq x < 0{\cdot}875$. For $0{\cdot}875 \leq x \leq 1$, $\bar{\Sigma}_y'$ is represented by the dotted curve.

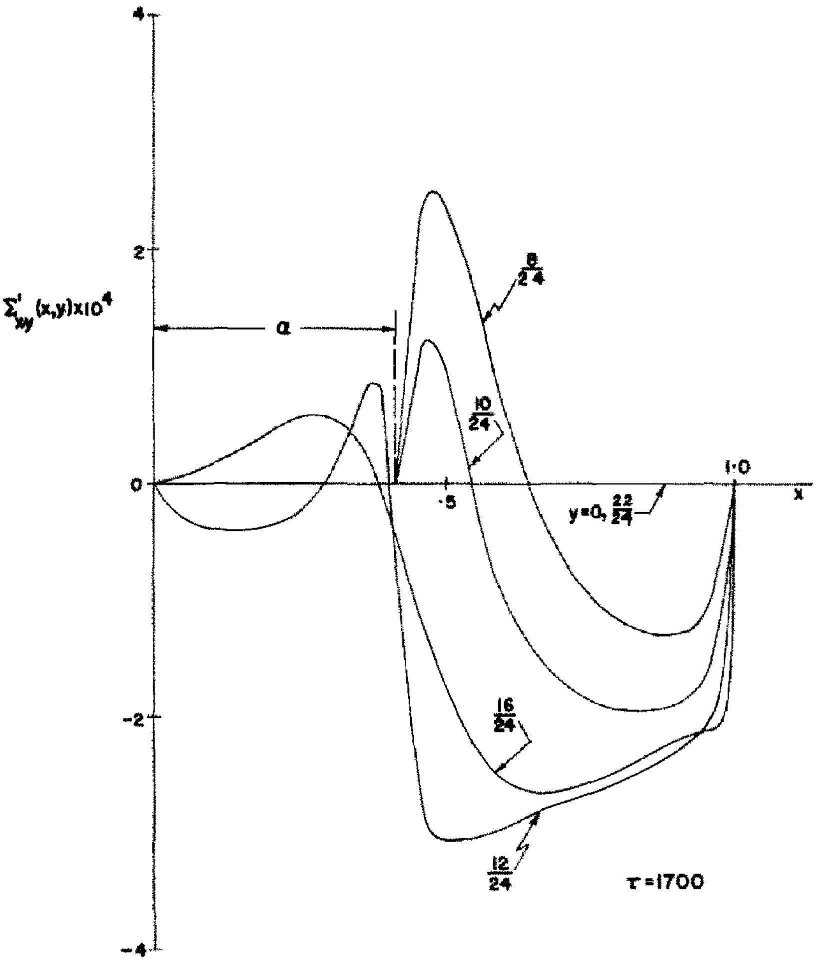

FIG. 7. The variation with x of Σ'_{xy} for Shell I for a sequence of y values.

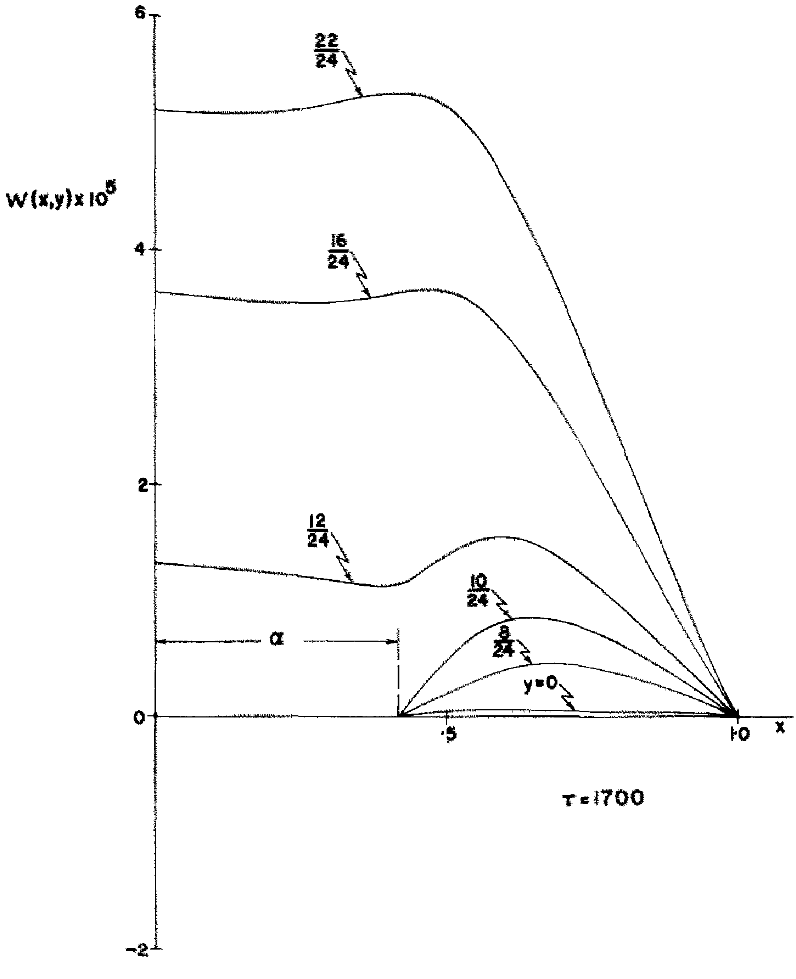

Fig. 8. The variation with x of the dimensionless normal displacement w for Shell I for a sequence of y values.

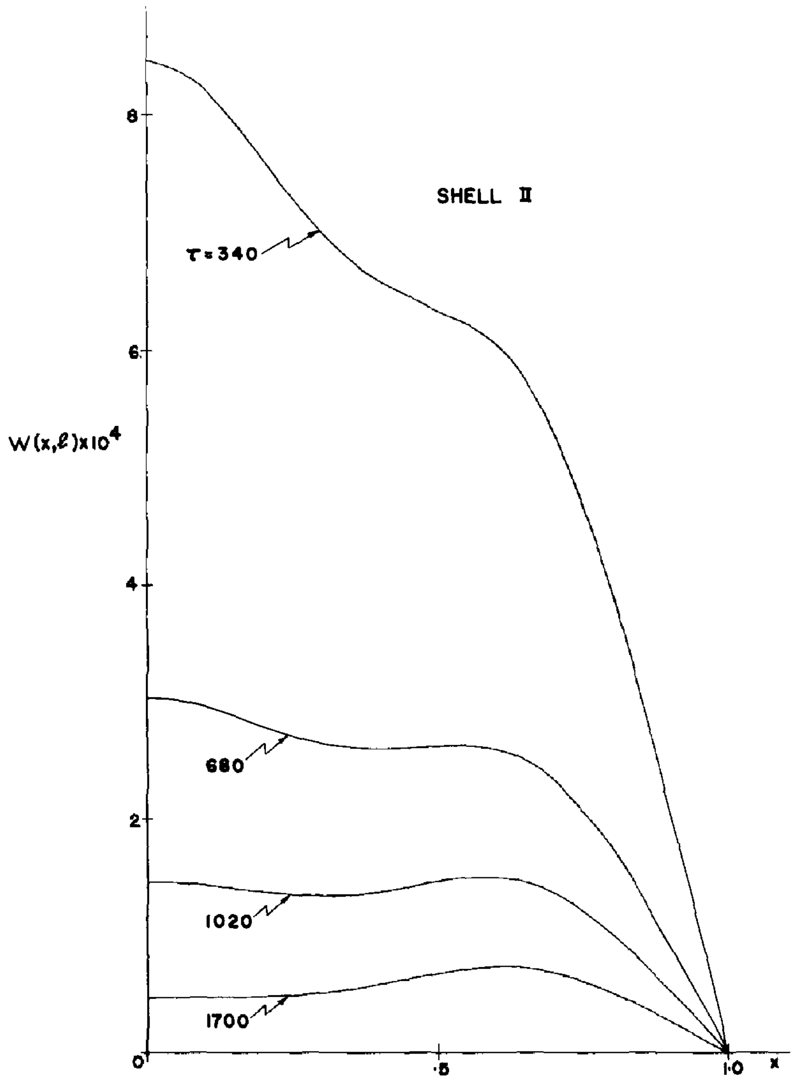

FIG. 9. The variation with x of the dimensionless normal displacement w for Shell II on the line $y = l$ for a sequence of τ values.

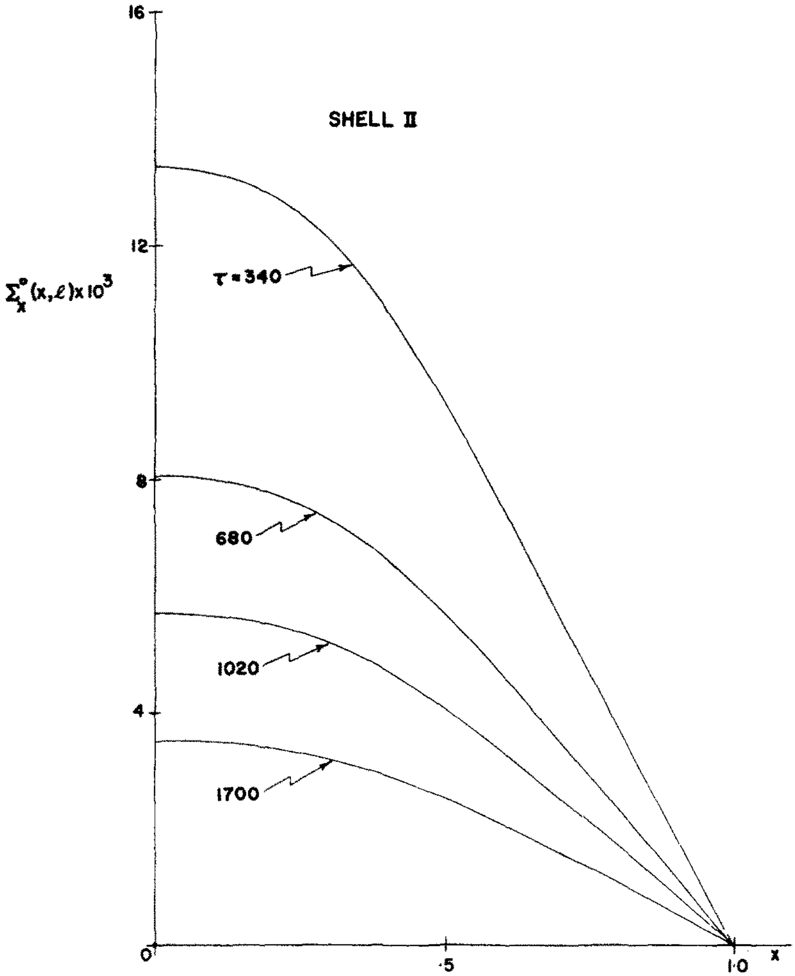

FIG. 10. The variation with x of Σ_x^0 for Shell II on the line $y = l$ for a sequence of τ values.

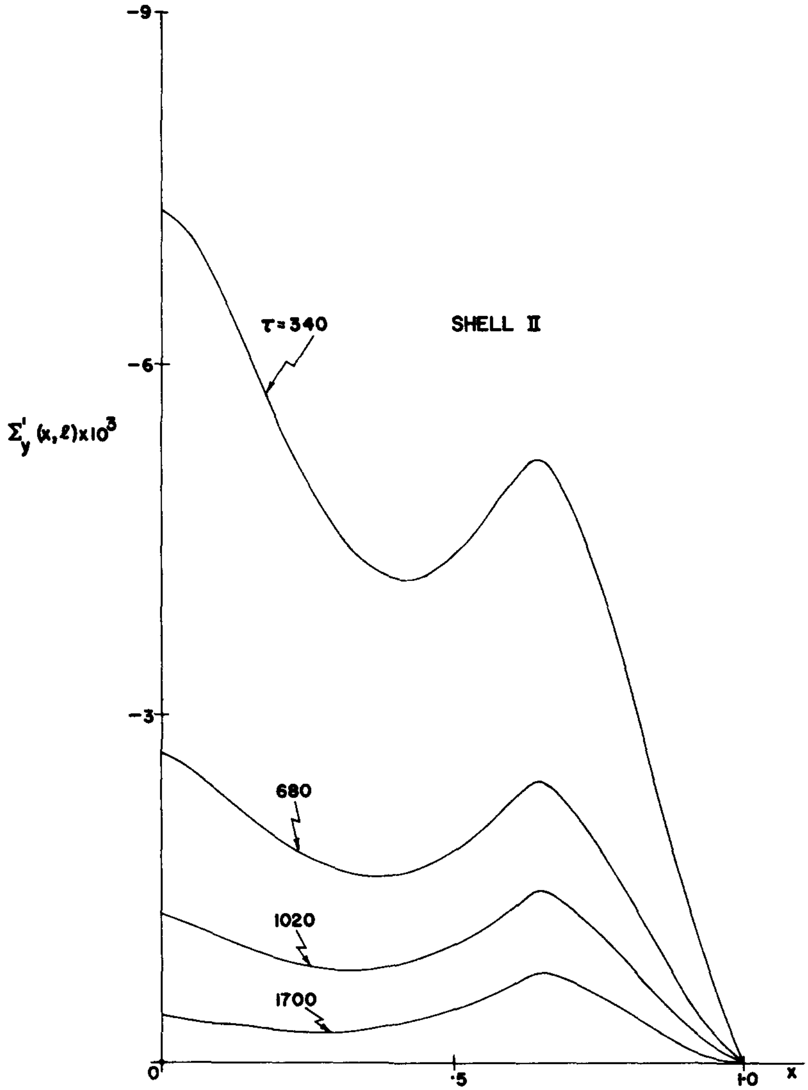

FIG. 11. The variation with x of Σ'_y for Shell II on the line $y = l$ for a sequence of τ values.

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Résumé—L'on a assumé que les déformations d'une enveloppe cylindrique élastique et mince, soumise à une pression latérale uniforme, sont adéquatement décrites par les équations de Donnell. La surface de l'enveloppe est perforée par un rectangle.

Les déplacements tangentiels de la membrane ainsi que les efforts normaux de membrane disparaissent sur les rebords simplement supportés du trou et sur les extrémités du cylindre. Des solutions numériques sont obtenues en remplaçant le problème à valeurs limites par une approximation de différences finies et en résolvant le système résultant d'équations algébriques linéaires par une méthode d'iteration.

Zusammenfassung—Es wird angenommen dass die Verformungen einer dünnen zylindrischen Schale, welche einem gleichmässigen Seitendruck unterworfen ist, durch die Donnell Schalengleichung hinreichend beschrieben sind. Die Oberfläche der Schale ist mit einem rechteckigen Ausschnitt perforiert. Die tangentiellen Membranverschiebungen und die üblichen Membranbeanspruchungen verschwinden an den einfach unterstützten Rändern des Loches und den Enden des Zylinders. Zahlenmässige Auflösungen werden erhalten durch Ersetzung des Randwertproblemes durch eine endliche Differenzenannäherung und Lösung des resultierenden Systems von linearen algebraischen Gleichungen durch ein Iterationsverfahren.

Абстракт—Принимается, что деформации тонкой цилиндрической упругой оболочки, подвергшейся равномерному поперечному давлению соответственно выражены уравнениями оболочки Доннелла (Donnell). Поверхнрсть оболоуки просверлена прямоугольным вырезком. Тангенциальные (поперечные) перемещения мембраны исчезают на обычно поддерживаемых краях отверстия и на концах цилиндра. Получены числовые решения замещением проблемы краевой задачи приближением по конечным разносцям и разрешая полученную в результате систему линейных алгебраических уравнений процессом повторения.